Velocity-dependent transverse momentum distribution of projectile-like fragments at 95 MeV/u

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r280n collaboration

Introduction
Investigation of unstable nuclei

Inverse kinematics scheme

Preparation of radioactive nuclear beams (RNBs)
1) Wide range in nuclear chart
2) High quality as a beam

Fragmentation process

Participant-spectator picture

Satisfies two requirements
1) High productivity for wide range of isotopes
2) High beam quality

→ Usually applied at RNB facilities
Production of a wide range of isotopes

Fragments prod. from $^{124}\text{Sn}+^{124}\text{Sn}$ at 1 AGeV

Produced isotopes

High quality as secondary beam

Momentum distribution of fragments
- $E$ dissipation: low
- Narrow width
- Simple & well studied at relativistic $E$

Therefore
- Separation of objective isotope with high efficiency

$P_L$ distribution of $^{10}\text{Be}$ produced from $^{12}\text{C}$ (2.1 GeV/nucleon)$+^{9}\text{Be}$

$P_0=2880$ MeV/c/A
Formulation for practical use

Transportation/separation of fragments through fragment separator

Performance of separator is simulated by means of LISE++, MOCADI, ...

Ex. \(^{40}\text{Ca}\) (80 AMeV) + Be optimized for \(^{32}\text{Mg}\)

http://lise.nscl.msu.edu/introduction.html

Key parameters for simulation

\(\sigma_{\text{prod.}}, P_{L}, P_{T}\) distribution

Few systematic studies on \(P_{T}\)

Object of this talk

Remarkable and systematic correlation between \(P_{T}\) distribution and \(P_{L}\) of fragmentation products at \(E\sim 100\) MeV/u

1. Earlier works on \(P_{L}, P_{T}\) distributions
2. Experimental (RIPS-RIKEN)
3. Correlation obtained from experimental results
4. Comparison with microscopic dynamic model
5. Conclusions
Previous studies on momentum distributions

Isotropic distribution at relativistic $E$

$E \gtrsim 1$ GeV/u

- Gaussian-type distribution with small energy dissipation
- Isotropic distribution $\sigma_L = \sigma_T$ to an accuracy of 10%

→ Contribution of Fermi momentum
Model based on Fermi momentum

Assumption:
Independent removal of nucleons in projectile

Momentum distribution of fragments corresponds to statistical sum of Fermi momentum for each removed nucleon.

Formulation proposed by Goldhaber

\[ \sigma_{GH}^2 = \frac{A_F(A_p - A_F)}{A_p - 1} \sigma_0^2 \]

\[ \sigma_0 = \frac{P_F}{\sqrt{5}} \sim 100 \text{ MeV/c} \]


Simple and successfully applied to a wide range of reaction system

Success of Goldhaber model

Width of \( P_L \) distribution of fragments
\( ^{36}\text{Ar}(1.05 \text{ GeV/nucleon}) + ^9\text{Be} \)

\[ \sigma_0 = 98.2 \text{ MeV/c} \]

\[ \frac{P_F}{\sqrt{5}} \sim 100 \text{ MeV/c} \]
Deviation from isotropic dist. at \( E \sim 100 \text{ MeV/u} \)

**\( \mathcal{P}_L \) distribution : Low momentum tail**

Universal parametrization obtained from experimental results


\[
f(P_L) = \exp\left(\frac{P_L}{\tau}\right) \left[1 - \text{erf}\left(\frac{P_L - P_0 + \sigma_{p_f}^2 / \tau - s \cdot \tau}{\sqrt{2} \sigma_{p_f}}\right)\right]
\]

\[
\tau = \text{coef} \cdot \frac{\sqrt{A_T \cdot E_S}}{\beta}
\]

\[
\sigma_{p_f} = \beta \sigma_{p_f}^2 \frac{A_F (A_F - 1)}{A_P - 1}
\]

Formulated momentum distributions have been incorporated into simulation.

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Deviation from isotropic dist. at \( E \sim 100 \text{ MeV/u} \)

**\( \mathcal{P}_T \) distribution : Additional width**

Additional width due to orbital deflection by target nucleus

\[
\sigma_T^2 = \sigma_{GH}^2 + \sigma_D^2
\]

Empirical formulation for \( \sigma_D \)

\[
\sigma_D^2 = \frac{A_F (A_F - 1)}{A_P (A_P - 1)} \sigma_2^2
\]

Usually applied with \( \sigma_2 \sim 200 \text{ MeV/c} \).

However, the reliability of the formulation is doubtful

- scarce systematic measurements at \( E \sim 100 \text{ MeV/u} \)
- no measurements as a func. of \( \nu \)

suggested from asymmetric \( \mathcal{P}_T \) dist.
Velocity dependent $P_T$ distribution

Angular dist. observed at 44 MeV/u

$^{40}\text{Ar} + ^{58}\text{Ni} \rightarrow ^{21}\text{Ne} + X$

- $\nu > 0.25 \, c$
- $\nu < 0.25 \, c$

Expected trend for width of $P_T$ distribution

Contribution of collective effect was considered to understand $\nu$-dep. angular distribution.


It is suggestive, but further investigations are needed to
1) formulate $\nu$-dependent behavior
2) understand based on reaction process.

Experimental

S. Momota  NUFRA2015, Oct/09/2015
Experimental : fragment separator
RIKEN Ring cyclotron + RIPS

Reaction : $^{40}\text{Ar} \ (95 \text{ MeV/u}) + ^9\text{Be}$

Identification : TOF, $\Delta E$, $B\rho$

Experimental : fragment separator
RIKEN Ring cyclotron + RIPS

Reaction : $^{40}\text{Ar} \ (95 \text{ MeV/u}) + ^9\text{Be}$
Experimental : fragment separator
RIKEN Ring cyclotron + RIPS

Reaction : $^{40}$Ar (95 MeV/u) + $^9$Be

Identification : TOF, $\Delta E$, $B_p$

Acceptance : Slit after target, F1

$\Delta P/P_0 = \pm 0.5\%$
$\Delta \theta_x, \Delta \theta_y = \pm 7.5$ mrad

Beam intensity monitor : PL@target
Ambiguity of beam intensity : $\sim \pm 5\%$
Experimental: fragment separator
RIKEN Ring cyclotron + RIPS

Reaction: $^{40}\text{Ar}$ (95 MeV/u) + $^9\text{Be}$
Identification: TOF, $\Delta E$, $B_p$
Acceptance: Slit after target, F1
Beam intensity monitor: PL@target
Ambiguity of beam intensity: $\sim \pm 5\%$
Angular distribution:
Beam swinger + slit after target
Keep optical axis of RIPS at any angle setting
$\rightarrow$ Constant values for transmission and detection efficiencies of fragmentation products.

Results & Analysis
Width of $P_T$ distribution

Obtained from angular distribution

$$B_\theta = 3.600 \text{ Tm}$$

Fitting with a Gaussian function

$$N(\theta) = A \exp \left\{ -\frac{\theta^2}{2\sigma_\theta^2} \right\}$$

with considering

1) Finite angular acceptance
   $\pm 7.5 \text{ mrad}$

2) Angular struggling in target
   evaluated by ATIMA

3) Emittance of primary beam
   assumed to be neglected

Width of $P_T$ distribution

$$\sigma_T = P_L \times \sigma_\theta$$

Correlation between $\sigma_T$ and velocity

40 Ar + Be $\rightarrow$ 26 Ne + X

$\Delta P$ : shift from primary beam velocity

$P_L$ distribution

Deceleration : $\sim 300 \text{ MeV/c}$

Larger width for low $P_L$

Remarkable decreasing trend

Agreement with reference values

GH@primary beam velocity
GH+Bibber@center of $P_L$ dist.
Correlation between $\sigma_T$ and $\Delta P_L$

Universal behaviors of $\sigma_T$

$A_F = 10 \sim 21$

$A_F = 22 \sim 33$

Fitting by a linear function: $\sigma_T = k_0 + k_1 \Delta P_L$

$\sigma_T$ at projectile velocity: $k_0$

Good agreement with Goldhaber formulation
No additional dispersions are not needed.
Reduced width : $\sigma_0$

$$\sigma_{GH}^2 = \frac{A_F(A_p - A_F)}{A_p - 1} \sigma_0^2$$

Av. = $93.6 \pm 1.3$ MeV/c

93.5 ± 2.6 MeV/c

obtained from $P_L$ dist.


Good agreement with $\sigma_0$ obtained from $P_L$ dist.

$\sigma_T$ at center of $P_L$ distribution

In order to compare with the previous results, most probable $\sigma_T$, $<\sigma_T>$, is introduced.

$<\sigma_T>$ : $\sigma_T$ at center of $P_L$ distribution

$\sigma_T$ at $E=92.5$ MeV/u

Consistent with previous results on $\sigma_T$

Slope parameter: $k_1$

Fitting by a quadratic function:
$$k_1 = -0.384 + 0.0273A_F + 0.000631A_F^2$$

Empirical formulation of $\sigma_T$

Width of $P_T$ distribution: $\sigma_T$
Monotonically decreasing with velocity
$$\sigma_T = k_0 + k_1 \Delta P_L$$

$\sigma_T$ at projectile velocity: $k_0$
$$\sigma_T(\Delta P_L=0) = \sigma_L = \sigma_{GH}$$

Slope parameter: $k_1$
Depends on $A_F$
$$k_1 = -0.384 + 0.0273A_F + 0.000631A_F^2$$

Microscopic reaction model
can reproduce behaviors of $\sigma_T$?
Can reveal origin of the behaviors?
Microscopic reaction model

Collective features

-> Additional dispersion and deceleration effect

Abrasion, Excitation

AMD

$^{40}\text{Ar}(87.4 \text{ MeV/A}) + ^9\text{Be}$

$b = 0 \sim 12 \text{ fm}$

Gogny type int.


Evaporation

Statistical decay


$P_T$ distribution obtained by simulation

$^{40}\text{Ar} + ^6\text{Be} \rightarrow ^{25}\text{Mg} + X$

$P_T$ distribution obtained from AMD + SD calculation

Gaussian-like distribution

Width is consistent with conventional values

Fitting with a Gaussian function as for experimental results

$\rightarrow \sigma_T(\text{AMD})$
Correlation between $\sigma_T$ and $\Delta P_L$

For $A_F = 10 \sim 21$:

- $\sigma_T(\text{MeV/c})$
- $\Delta P_L(\text{MeV/c})$

For $A_F = 22 \sim 33$:

- Remarkable agreement for $A_F = 30 \sim 33$

AMD calculation roughly reproduces behaviors of $\sigma_T$.

Fitting with a linear function $\rightarrow k_0, k_1$

$\sigma_T$ at projectile velocity: $k_0$

Not so bad, but systematically underestimate $k_0$ at $A_F > 30$. 

AMD+SD

- GH
- GH + Bibber
- B, C
- N, O
- F, Ne
- Na, Mg
- Al, Si
Slope parameter: $k_1$

Parabolic trend of $k_1$ obtained from experimental results

AMD calculation roughly reproduces negative values for $k_1$. Large scattering prevents further investigations on $k_1$.

$b$-dependent $P_L$ distribution

Impact parameter $\leftrightarrow$ Collective/dissipative nature

Therefore, $P_L$ distribution is expected to depend on impact parameter.

Prod. rate of $^{30}$Si vs. $b$

$r^{(40)Ar} + r^{(9)Be} = 6.6$ fm

$r = r_0 A^{1/3}$, $r_0 = 1.2$ fm


$E$-dissipation is promoted for small $b$. 

Contribution of impact parameter

\( b \)-dependent \( P_L \) distribution

\[ \frac{d\sigma}{d\Delta P_x \Delta P_y} \text{(mb/(MeV c}^{-1} A^{-1})^2) \]

\( \Delta \sigma/\Delta b \text{ (mb/fm)} \)

\( b \text{ (fm)} \)

- High \( P_L \) component
  - Contribution of larger \( b \) is dominant.

- Low \( P_L \) component
  - Contribution of smaller \( b \) is dominant.

Produced rate vs. \( b \)

\( ^{40}\text{Ar} + ^{9}\text{Be} \rightarrow ^{30}\text{Si} + X \)

\( b \)-dependent \( P_T \) distribution

The width of \( P_T \) distribution is larger for small \( b \).

\( \rightarrow v \)-dependent \( \sigma_T \) would be originated from contribution of impact parameters.
**$P_T$** distribution at higher energy

**$E = 95$ MeV/u**

$^{40}$Ar + Be $\rightarrow$ $^{26}$Ne + X

![Graph showing $P_T$ distribution at 95 MeV/u](image)

**$E = 290$ MeV/u**

$^{40}$Ar + Al $\rightarrow$ $^{26}$Na + X

![Graph showing $P_T$ distribution at 290 MeV/u](image)

Velocity dependence is not remarkable.

**$P_T$** distribution with heavier target

**Dominant contribution of repulsive Coulomb force**

![Diagram showing repulsive Coulomb force](image)

- Deflection effect was observed as off-centered $P_T$ distribution.
- Deflection is remarkable for small $\Delta A$ reaction.

Specified impact parameter can be defined for given isotope.

Possibility to investigation proximity potential for heavy reaction system

Conclusions

- Remarkable **correlation** between width of $P_T$ distribution ($\sigma_T$) and fragment velocity ($\Delta P_L$) has been observed at $E = 95$ MeV/u.

  1. Simple **formulation** : $\sigma_T = k_0 + k_1 \Delta P_L$

  2. Comparison with **previous results**

     $\sigma_T(\Delta P_L = 0) = \sigma_{GH}$

     Width of $P_L$ distribution

     $\sigma_T$ at center of $P_L$ dist. = $\sigma_D$

     Conventionally used value

  3. Important contribution of **impact parameter** to understand observed correlation

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